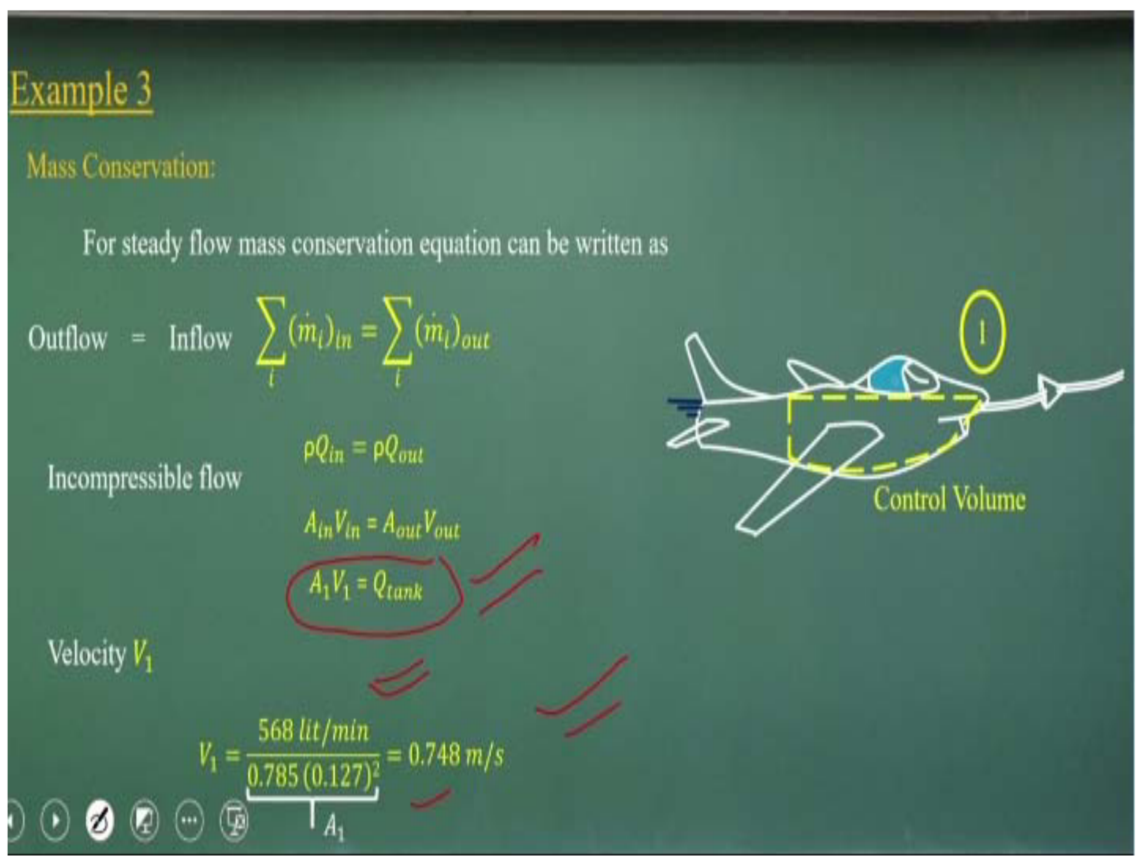

Now let us apply the mass conservation equation where is inflow minus outflow and this case will be,

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

Incompressible flow

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = Q_{tank}$

$$V_1 = \frac{568\ lit/min}{0.785\ (0.127)2} = 0.748\ m/s$$

This is what the discharge is given to us, the volumetric discharge is given to us divide by the area will get it the velocity, that the velocity part what we have.

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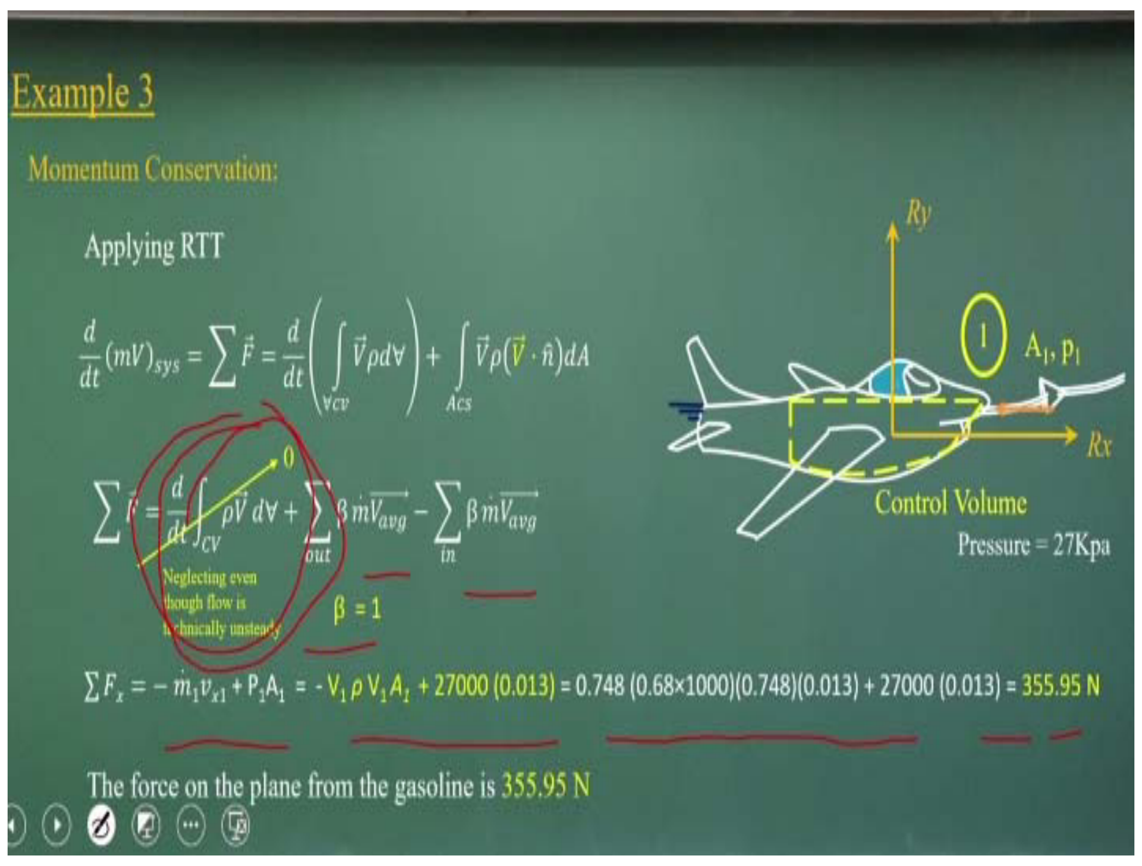

Now we have to find out what is the force acting in R_x and the R_y directions, we have the pressure component here. Applying this Reynolds transport theorems and the simplifying the tops. Here, this is the main concept what we have consider it that there is a change of the momentum flux within the control volume. But the right of the change of the momentum flux within a control volume, is not that significant order as compared with the momentum thrust what is coming from the fuel.

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Neglecting even though flow is technically unsteady, $\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall = 0$

$\beta = 1$

$$\sum F_x = -\dot{m}_1 v_{x1} + P_1 A_1$$

$$= -V_1\rho V_1 A_1 + 27000\,(0.013)$$

$$= 0.748\,(0.68 \times 1000)(0.748)(0.013) + 27000\,(0.013)$$

$$= 355.95\, N$$

So this component will be there, but it is not that significant. Since it is not significant we make it to zero and make it the problems in a steady nature. Then we have a two momentum flux components, one is influx another is outflux, beta is equal to the 1. So if I apply it, I will have a simply the momentum flux in x direction, the pressure into the area and that what I will get it and in terms of value, I will get it 355 approximately Newton.

That what the force will be acting on the x direction. Now take it another examples which are again simple examples what we are talking about, but is the control volume which is the moving conditions. That means moving control volume problems we will going to solve it.

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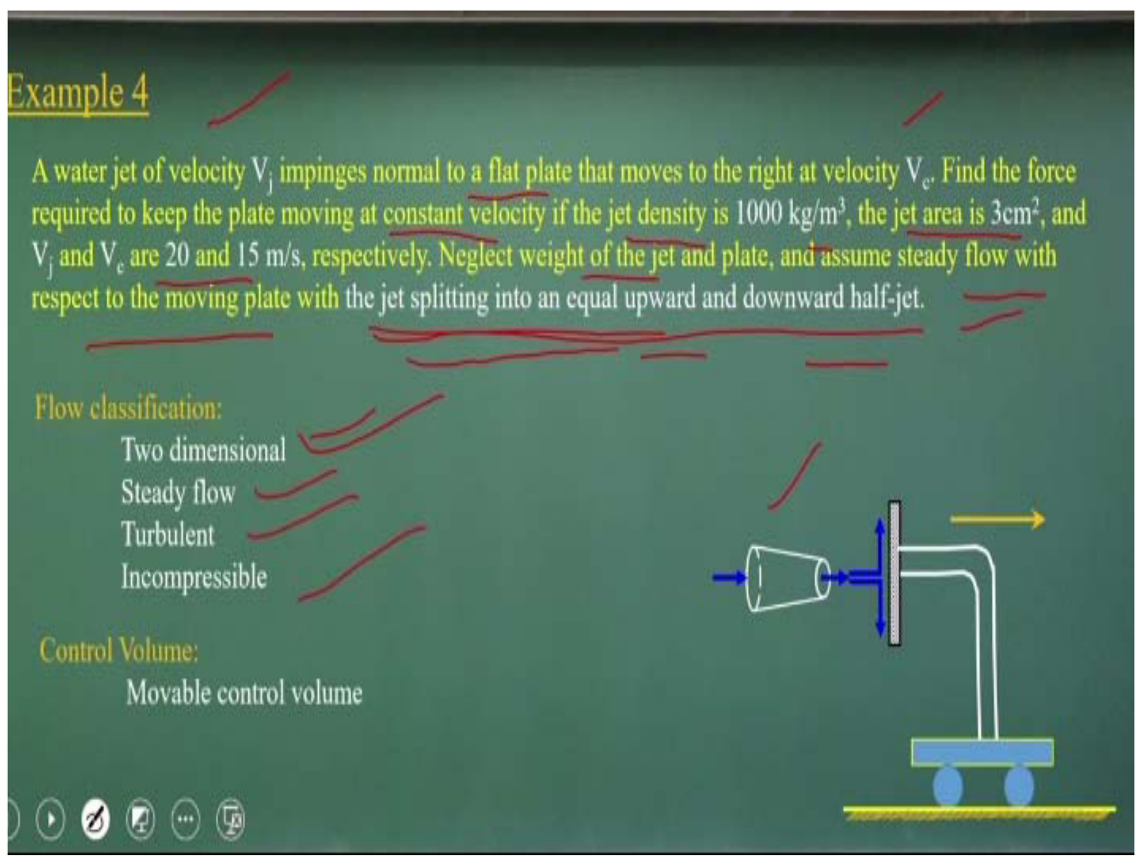

Like let we will have a water jet, the if you look it this is the water jet, is impinging this normal to a flat plate and moves to the right to the velocity V c, okay this that means, this flat forms is moving with a velocity V c. Find the force required to keep the plate moving at the constant velocity if jet density is this, the water density is this. Jet area is given and V j the velocity of water jet and the V c the velocity of the frame is respectively 20 and 15 meter per second.

[A water jet of velocity V_j impinges normal to a flat plate that moves to the right at velocity V_c. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m^3, the jet area is 3cm^2, and V_j and V_c are 20 and 15 m/s, respectively. Neglect weight of the jet and plate, and assume steady flow with respect to the moving plate with the jet splitting into an equal upward and downward half-jet.]

Then neglect the weight of the jet and the plate, assume the steady flow with respect to moving plate. Jet is splitting into two equal upward downward half-jet. So these are the assumptions, quite valid for a jet flow problems is quite valid. So as the flow is impinging and making into two directions, vertical directions. One is jet is coming down, another is up.

Flow classification:

- Two dimensional
- Steady flow
- Turbulent
- Incompressible

You will have a two dimensional flow problems becomes a steady the turbulent and incompressible flow. Movable control volume concept only the things what we have here to know it that we will apply this movable control volumes to know it the pressure distribution part.

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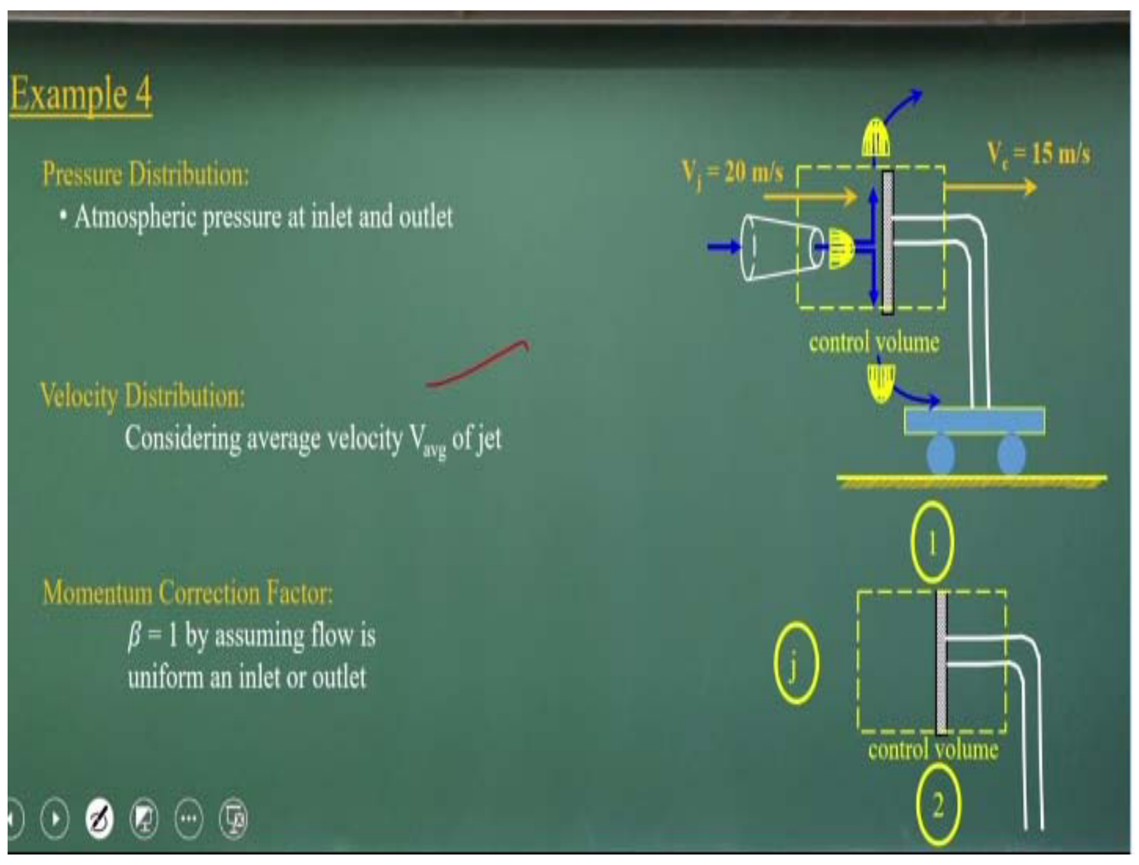

The pressure distributions in atmospheric pressure distributions will be the atmospheric pressure distribution the we will consider the average velocity conditions and the beta equal to the 1. Okay that the great simplifications we used to do it.

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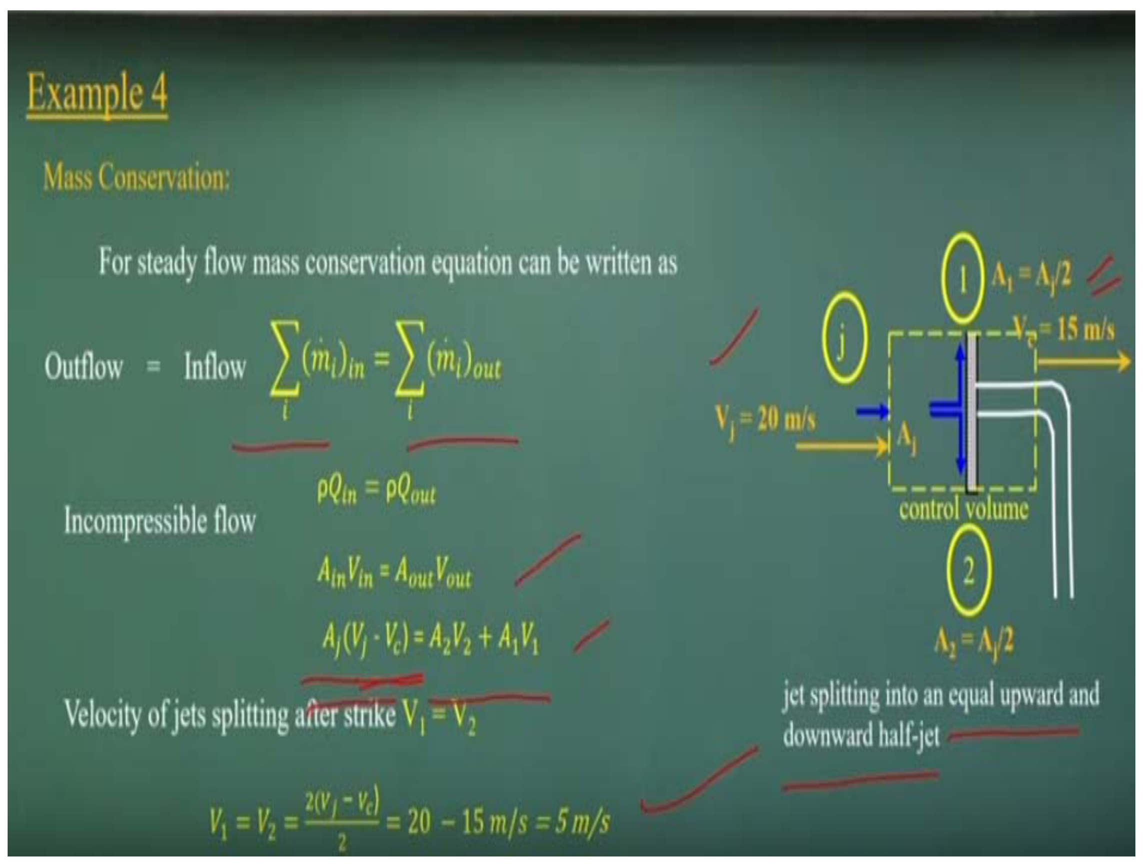
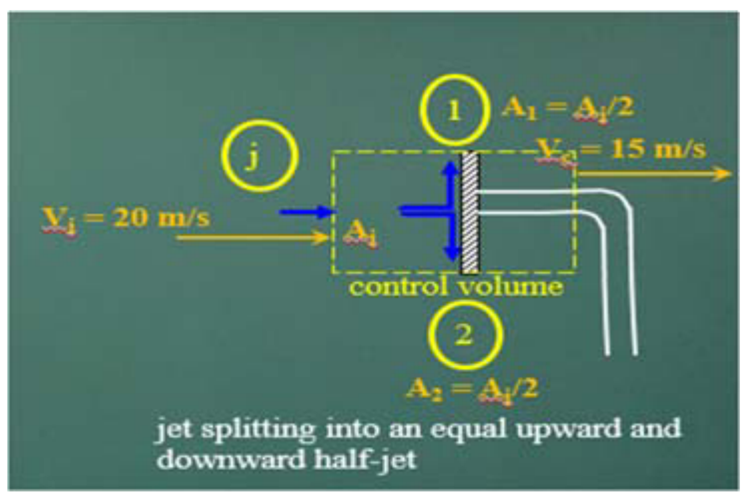


And if this is the V 1 to this cross section V z is water is coming into that and it is splitting into two part is equally part. So this is velocity of jet, this is what the moving control volume is moving with 15 meter per second. Jet is splitting into equally upward half-jet, the half of what goes to upward direction, half of water goes to the downward direction. If is that let us first apply the mass conservation equations.



The mass influx is equal to mass outflux. If that is the conditions you have a these values, okay. Please remember it we consider here the relative velocity component along the and it has the V_1, V_2 velocity which is going out where the velocity components, we do not have any velocity components in the y direction. But is only moving in the x direction, that is the reason.

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_j(V_j - V_c) = A_2V_2 + A_1V_1$

Velocity of jets splitting after strike $V_1 = V_2$

$$V_1 = V_2 = \frac{2(V_j - V_c)}{2} = 20 - 15\ m/s = 5\ m/s$$

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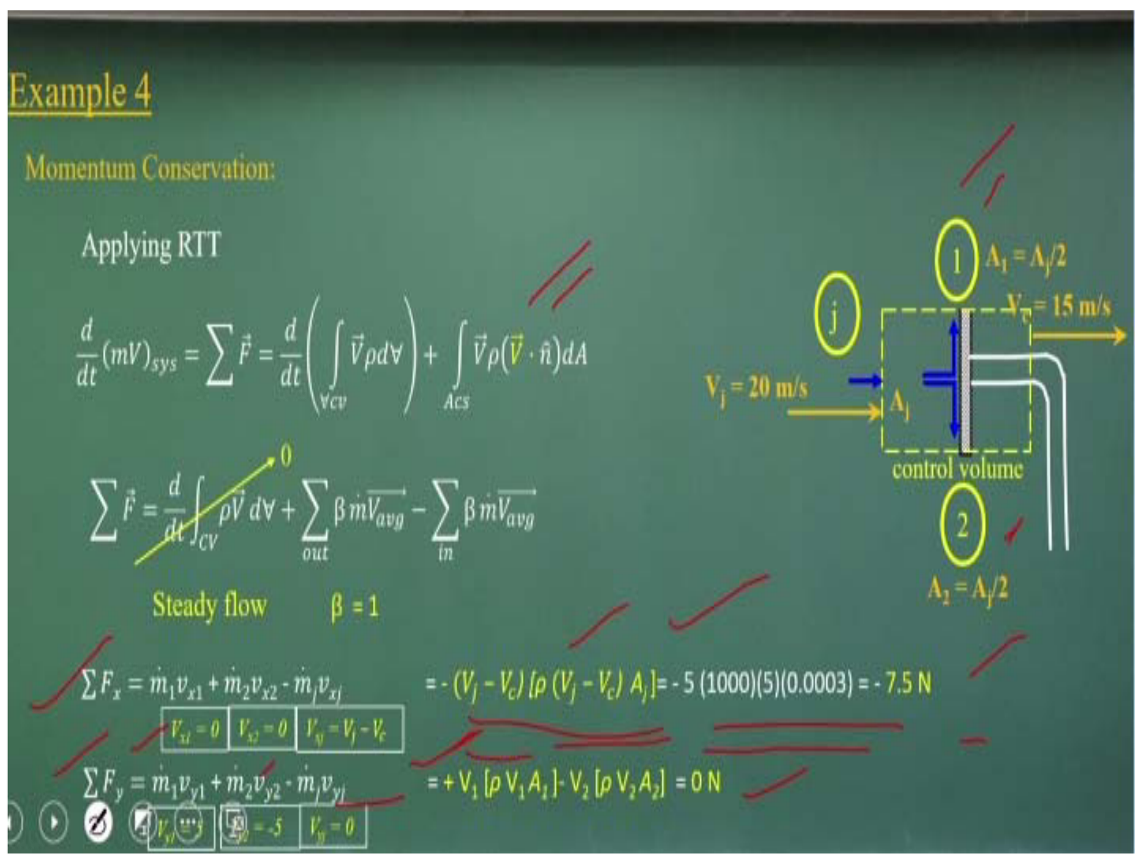

Now I am applying momentum conservation equation. We remember it I need to apply the relative velocity component here as the part the considerations here and beta equal to the 1 and I apply the relative velocity component. That what you can look it how the relative velocity components are used for the mass flux and the momentum flux component and that what is substitute to get it what is the force component. So the problem is similar nature.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

Only what we are doing it, we compute the relative velocity. The V xz which is V j – V c. V j is the velocity of jet minus V c is the velocity of the platforms moving with V c. That in the x direction. The flow what is going in the direction of 1 and 2 as you know it does not have any component in the x direction, we can make it zero.

So only the force we have because of the impinging of jet what is working with a relative velocity component that what we have substitute to finally we got it what is the amount of force acting because of these things. Similar way, the y direction also we can substitute it and apply it. As it expected it when you have a the same amount of water goes in the 1 and 2 the same velocities, they will be cancelled each others.

$$\sum Fx = \dot{m}_1 v_{x1} + \dot{m}_2 v_{x2} - \dot{m}_j v_{xj} = (V_j \text{ -} V_c)[\rho\ (V_j \text{ -} V_c)\ \ Aj\ ]$$

$$= -\,5\ (1000)(5)(0.0003)\ \ =\ -\,7.5\ N$$

So the net force will be act is zero, the force component will be the positive and the negative. The net momentum flux will be the zero. That the force component will come it.

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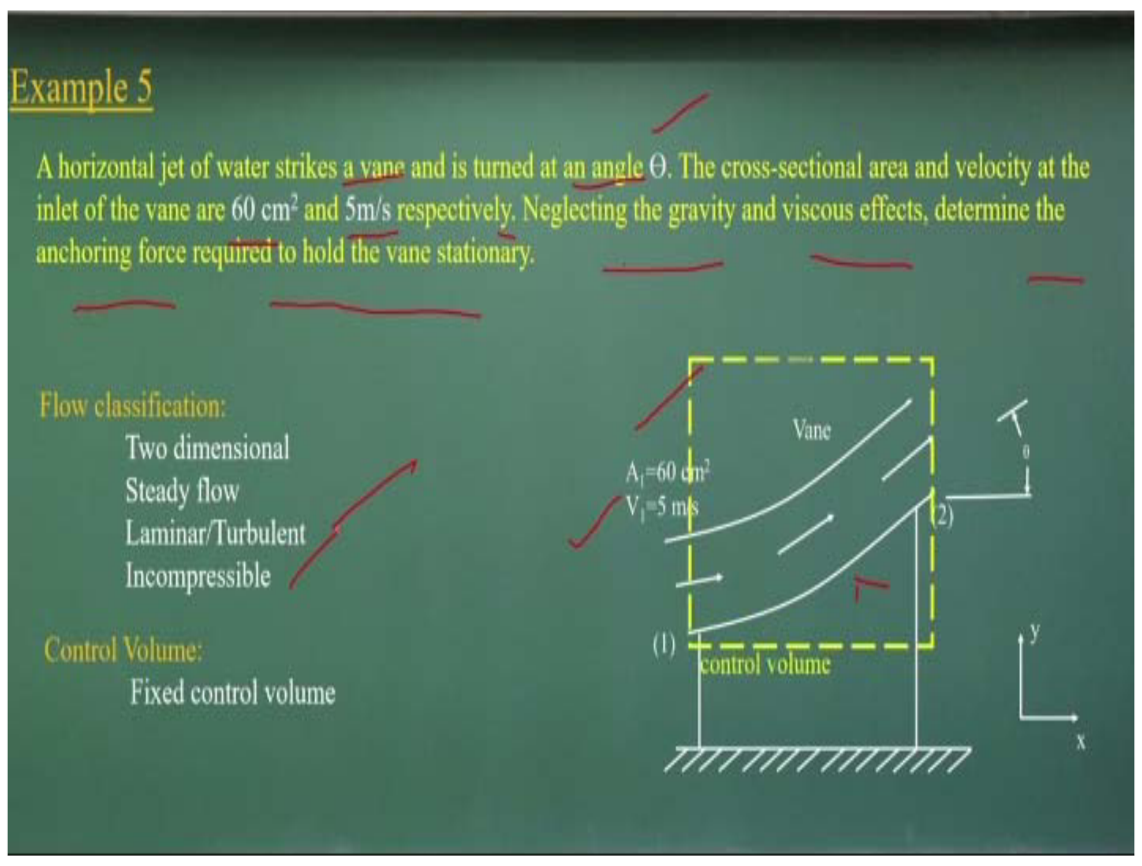

Now let us come to the example fifth which is again a simple problem where a horizontal jet of water strikes a vane, this is the vane, okay? Is turned the angle to theta degree. Cross-sectional area and velocity at the inlet of the vane is 60 centimeters, 5 meter per second respectively neglecting the gravity and viscous effect, okay?

[A horizontal jet of water strikes a vane and is turned at an angle Θ. The cross-sectional area and velocity at the inlet of the vane are 60 cm^2 and 5m/s respectively. Neglecting the gravity and viscous effects, determine the anchoring force required to hold the vane stationary.]

Determine this anchoring force required to hold this vane at the rest or the stationary. This is very simple problems okay. There is flow jet is coming. We are making angle theta which is coming out from this jet. The area is given, velocity is given and we need to compute it what amount of force is required to anchor this vane so that the vane will be at the rest conditions or the stationary will be there.

Flow classification:

Two dimensional

Steady flow

Laminar/Turbulent

Incompressible

Flow is two dimensional, steady. We do not know whether flow is a laminar or turbulent or incompressible flow. Fixed control volume. This is what the control volumes.

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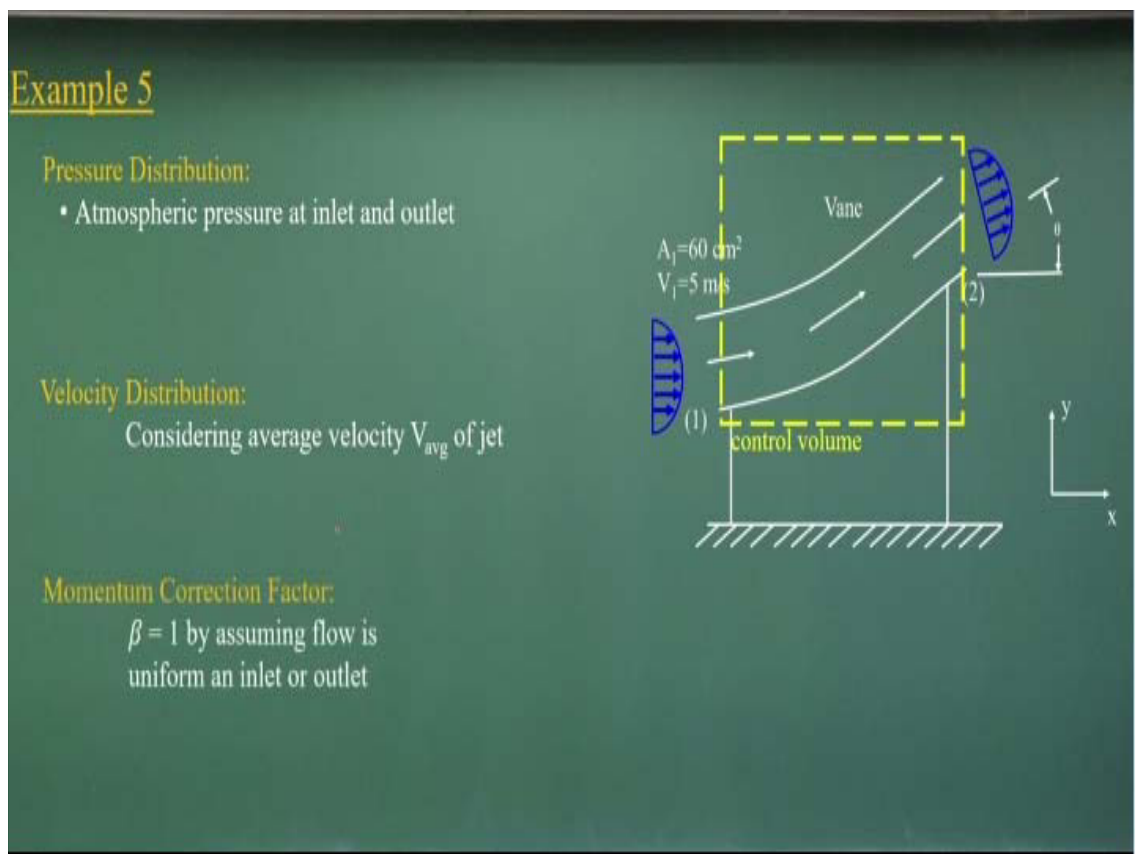

And the jet is there is atmospheric pressure and the velocity distribution again we can consider is a uniform. That is what not a conditions when you have the free jet in it going through the vane V but we can assume it the velocity distribution is uniform.

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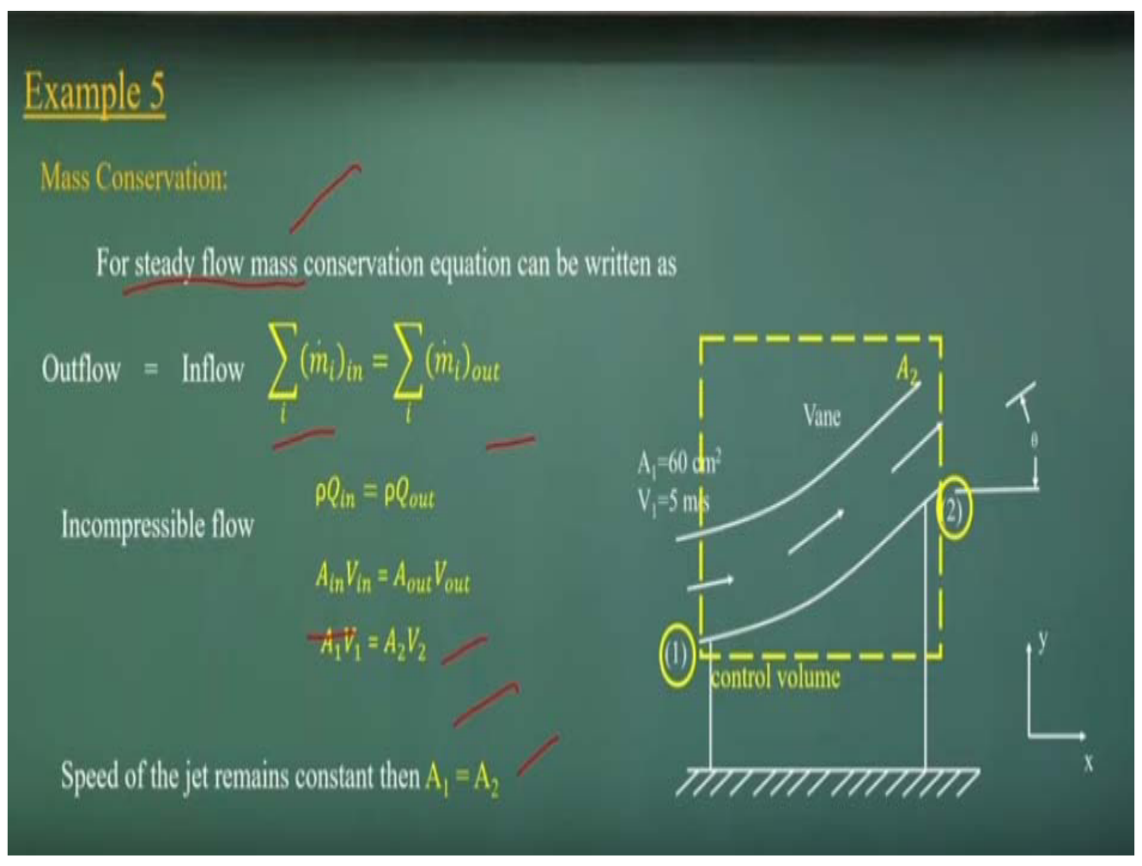

And if that is there we are computing the first mass conservation which is very easy things here being a steady flow we will have a mass influx is equal to this is a very standard things as we discussed earlier and that is what we will show it since the speed of the jet remains constant, the area of flow A 1 = A 2 following the continuity equations as the flow is incompressible.

Outflow = Inflow

$$\sum_i (\dot{m}_i)_{in} = \sum_i (\dot{m}_i)_{out}$$

Incompressible flow

$\rho Q_{in} = \rho Q_{out}$

$A_{in}V_{in} = A_{out}V_{out}$

$A_1V_1 = A_2V_2$

Speed of the jet remains constant then $A_1 = A_2$

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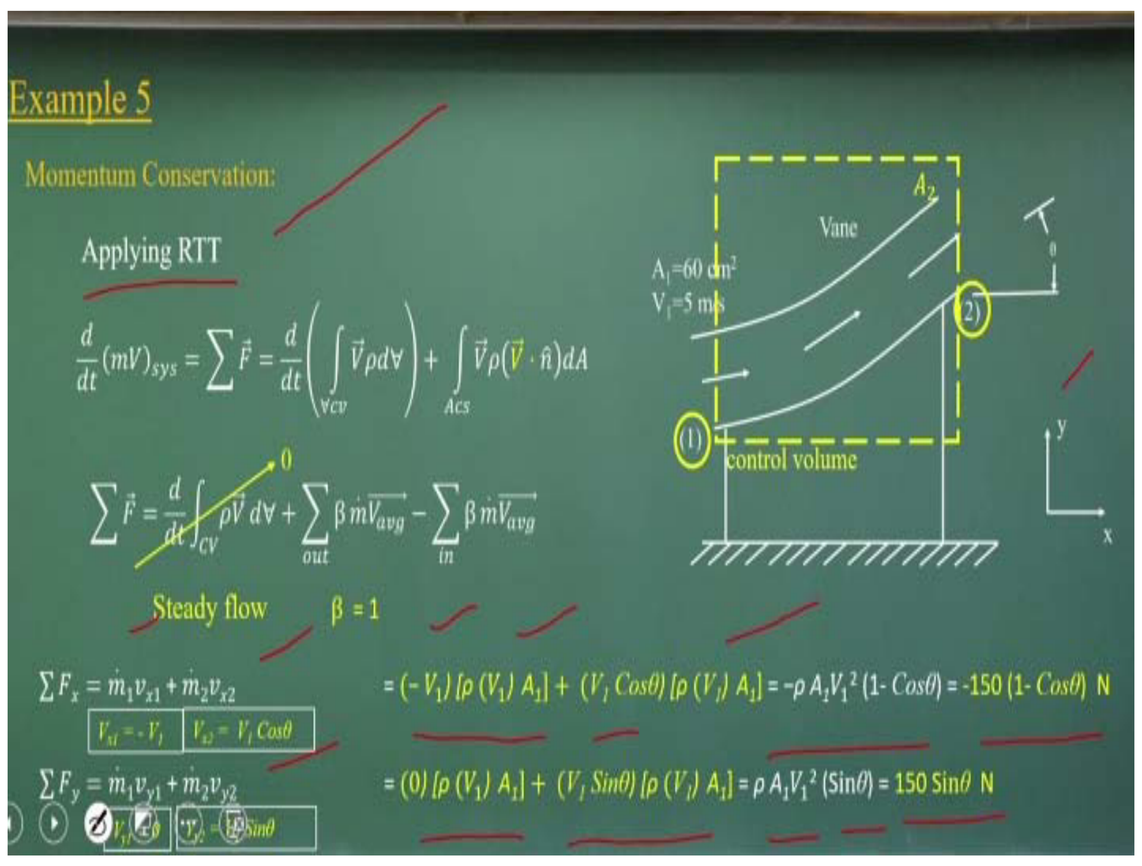

Now need to apply the momentum conservation equation, apply the Reynolds transport theorems to compute what will be the force. First again steady flow conditions which is standard assumptions we have been doing it for simplifying the problems for the problems here what we have considered it and β equal to 1.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\sum \vec{F} = \frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall + \sum_{out} \beta\, \dot{m}\overrightarrow{V_{avg}} - \sum_{in} \beta\, \dot{m}\overrightarrow{V_{avg}}$$

$$\frac{d}{dt}\int_{CV} \rho\vec{V}\, d\forall \text{ tends to } 0$$

$\sum Fx = \dot{m}_1 v_{x1} + \dot{m}_2 v_{x2} = (-V_1)\,[\rho\,(V_1)\,A_1] + (V_1\,Cos\theta)\,[\rho\,(V_1)\,A_1] = -\rho\,A_1 V_1^2\,(1-Cos\theta) = -150\,(1-Cos\theta)\;N$

$V_{x1} = -V_1$

$V_{x2} = V_1\,Cos\theta$

$\sum Fy = \dot{m}_1 v_{y1} + \dot{m}_2 v_{y2} = (0)\,[\rho\,(V_1)\,A_1] + (V_1\,Sin\theta)\,[\rho\,(V_1)\,A_1] = \rho\,A_1 V_1^2\,(Sin\theta) = 150\,Sin\theta\;N$

$V_{y1} = 0$

$V_{y2} = V_1\,Sin\theta$

The velocity component x direction and the mass flux that what we will equate it, it will be this value. Similar way if I am to compute the force acting in this y direction, I will have mass flux coming into momentum flux in the x y directions will be the zero. Influx is not there, it is coming imposing on the x direction.

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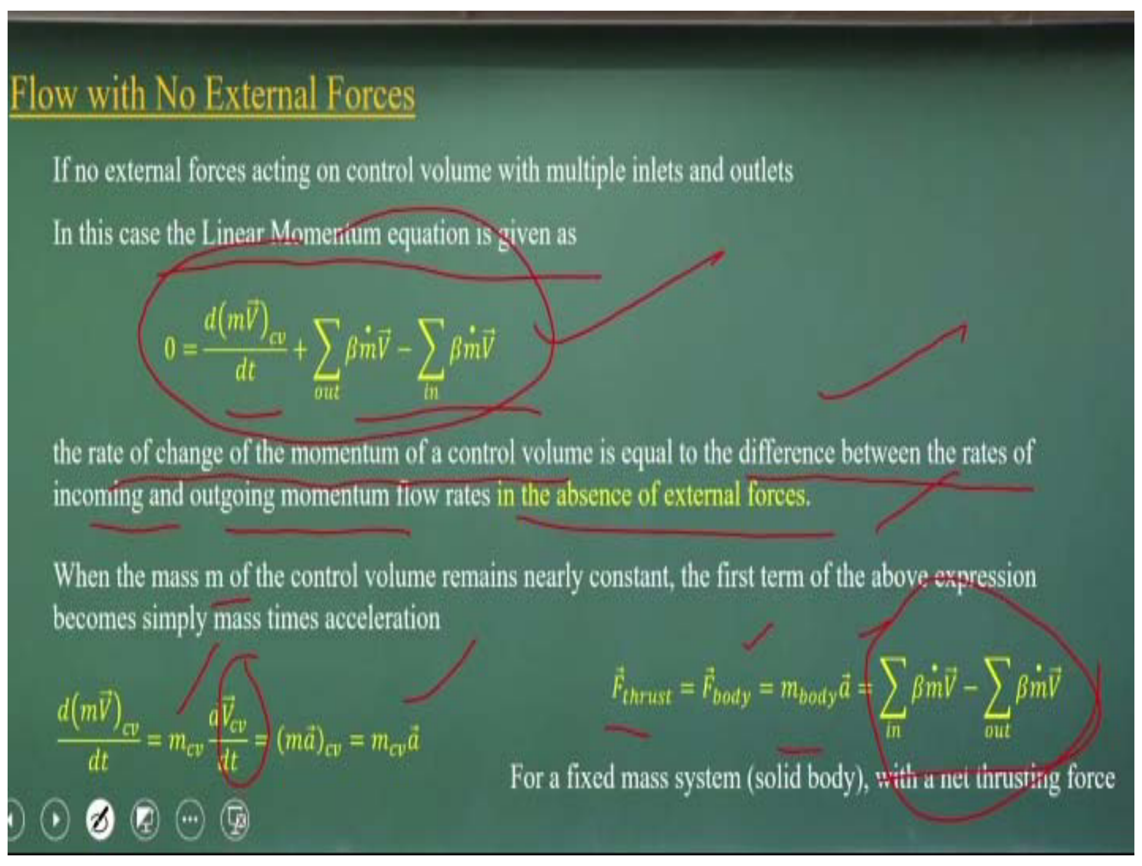

Now let us come it before ending this lecture, let us have a one examples very interesting examples of decelerating the spacecraft before landing on the earth. This is what very interesting problems. That means when you have a spacecraft there is no external force acting on that. And if you have that there is a multiple inlet and outlet. You can write the linear momentum equations sum of the force in the systems is equal to zero.

$$0 = \frac{d(m\vec{V})_{cv}}{dt} + \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$$

$$\frac{d(m\vec{V})_{cv}}{dt} = m_{cv}\frac{d\vec{V}_{cv}}{dt} = (m\vec{a})_{cv} = m_{cv}\vec{a}$$

You will have a these two point. The rate of the change of the momentum flux within the control volume is equal to net outflux of momentum flux passing through this control surface. That is what is written here. Rate of change of momentum, within a control volume is equal to the difference between the rates of incoming, outgoing momentum flux rate in absence of external force.

This is what the text, this is what mathematical explanations for this. So if you assume it is a mass of the control volume is m which remains constant. Then we can write this part is mass of control volume dV by dt. That is acceleration. So mass of control volume into the acceleration. The thrust of the force will going to act it is force acting on the body, the mass body into the acceleration.

$$\vec{F}_{thrust} = \vec{F}_{body} = m_{body}\vec{a} = \sum_{in} \beta \dot{m}\vec{V} - \sum_{out} \beta \dot{m}\vec{V}$$

That what net outflux of the momentum flux which is going out from this. This is very interesting problems. We will consider it when there is no external force acting on a control volume even if you have a multiple inlet and outlet.

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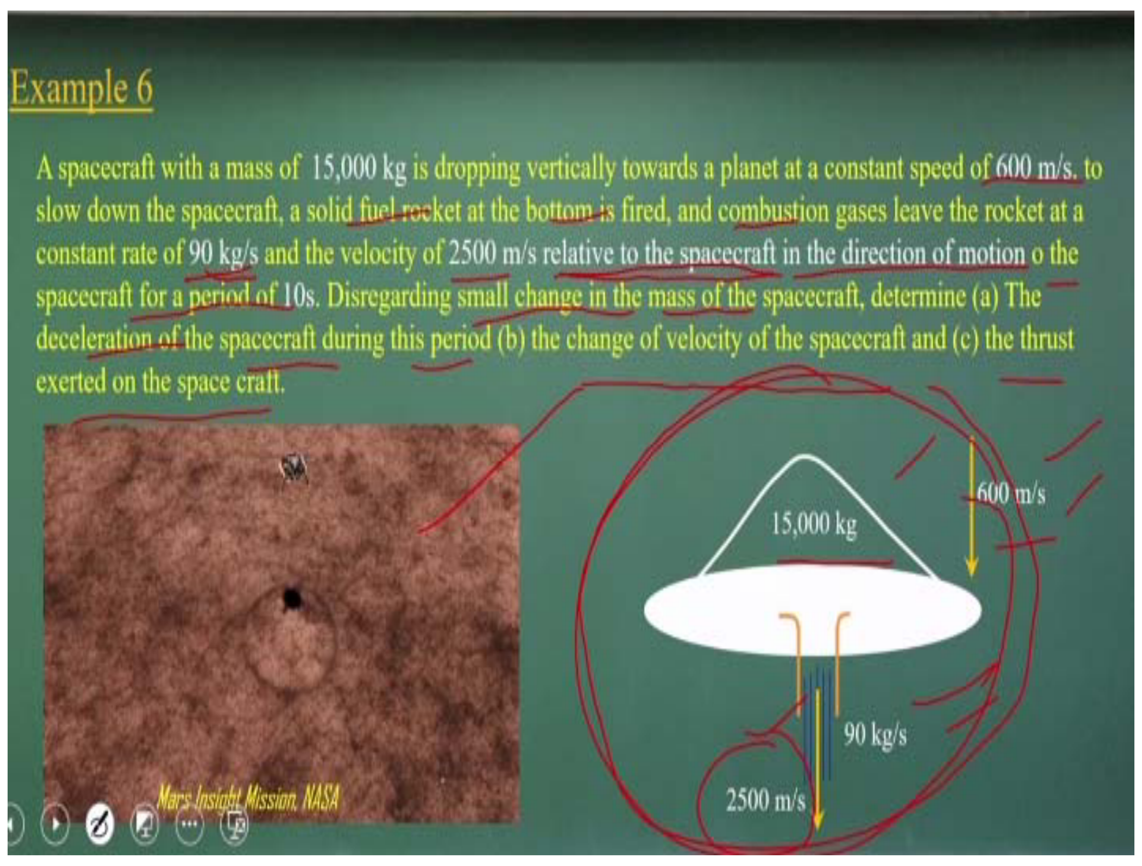

Let us see this, the figure what is showing is that when a spacecraft are landing on the surface, it decelerate it. It decelerate the terminal velocities and then it land on the earth or the any planets. So how do you do decelerate it? That is what we do it to decelerate that. It is the problems what you have consider it a spacecraft with a mass of 15000 kg dropping vertically towards a planet at a constant speed of 600 meter per second.

To slow down to spacecraft a solid fuel rocket at the bottom is fired. Combustion of gases leave the rocket at a constant rate of the speed of 90 kg per second. This is the what the mass rate is going up. The velocity will be 2500 meter per second relative to the spacecraft. Please remember this is what the main point, relative to spacecraft in the directions of the motions.

Then if that is what for the spacecraft for a period of the 10 second, this disregarding or neglecting the small change of the mass in the aircraft determine the deceleration of the spacecraft during this period, change of the velocity of the spacecraft and what is the thrust exerted on the spacecraft. So if you look it this is the spacecraft. This is what the Mars inside mission, how the diagrams what is from the NASA that is what we have illustrated.

Looking at these problem text whatever we discuss it the problems like this there is a spacecraft having 15,000 kg weight, the speed is 600 meter per second, which is injected a mass 90 kg per second, mass flow rate. And it has a relative velocity which 2500 meter per second. What is the amount of force acting due to that? What is the decelerating because of these firing of the gases from this rocket. And what is the change of the velocity that is what we will solve it.

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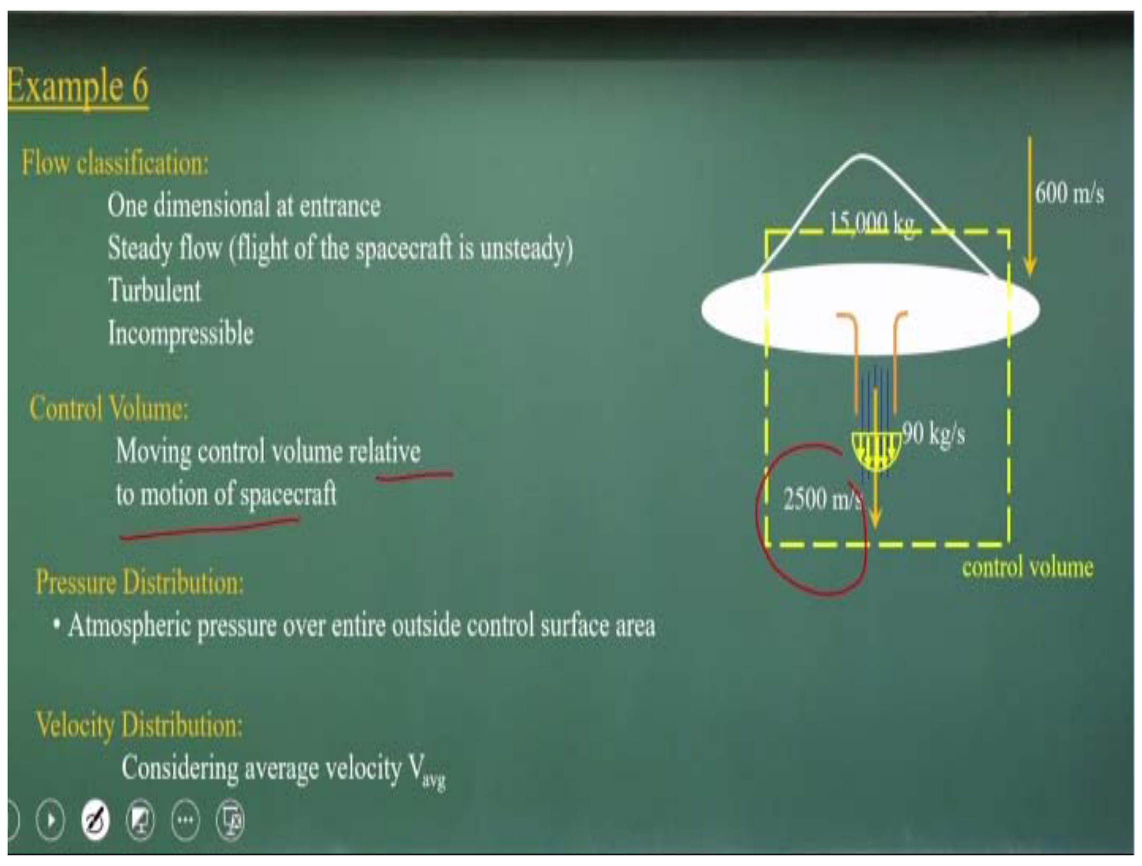

Now the problem is one dimensional because only one jet is there. We can use as a steady problems, turbulent, incompressible. This is a moving control volume, relative to motion of the space aircraft. Here we are talking about the relative velocities. We are not talking about the absolute velocity of jet and if it is that we consider the control volumes and try to find out what will be the force is acting because of this firing of the gases from this space aircraft.

Flow classification:

One dimensional at entrance

Steady flow (flight of the spacecraft is unsteady)

Turbulent

Incompressible

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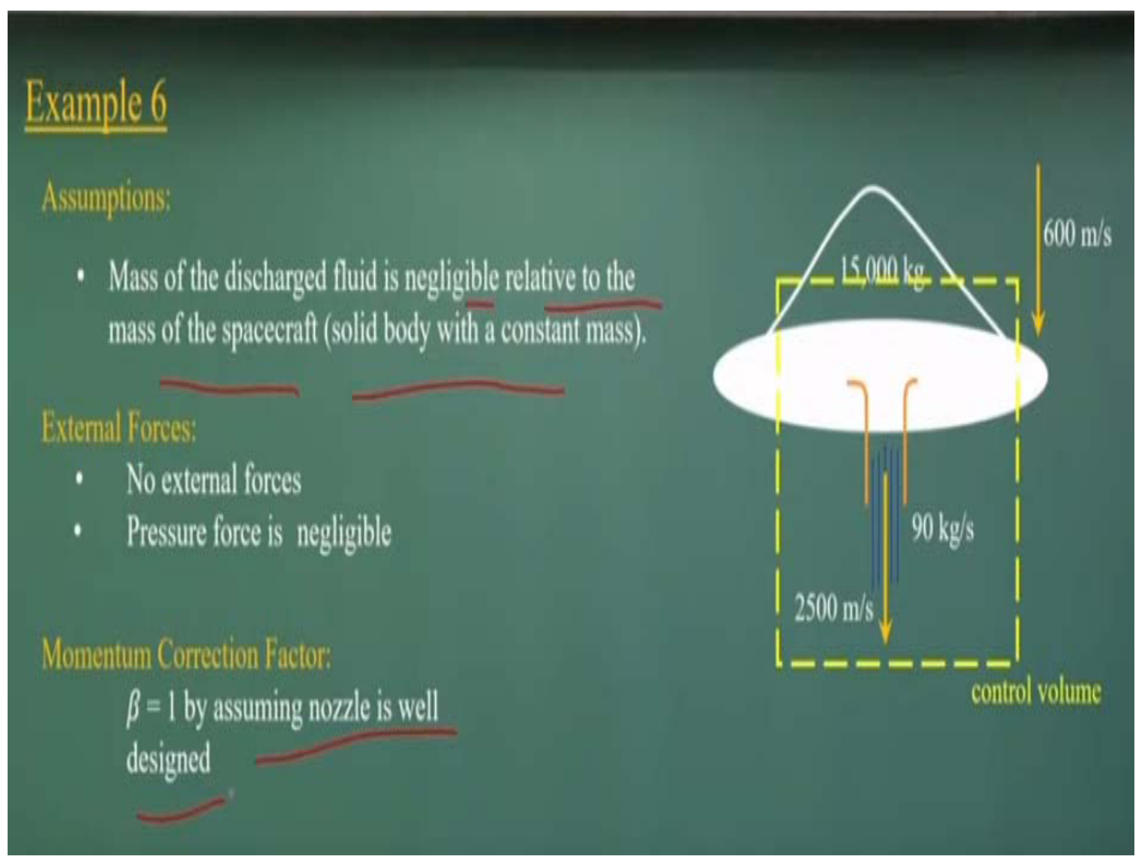

Now if I have a another assumption as you said, the mass of the discharged fluid is negligible, is relative to the mass of the aircraft. This will be the solid bodies moving with a constant mass, okay. No external forces. The pressure forces can be negligible. Beta is equal to the 1, assuming the nozzle is well designed, okay. So because a well designed nozzle have a close to the uniform velocity distribution. So beta will be 1.

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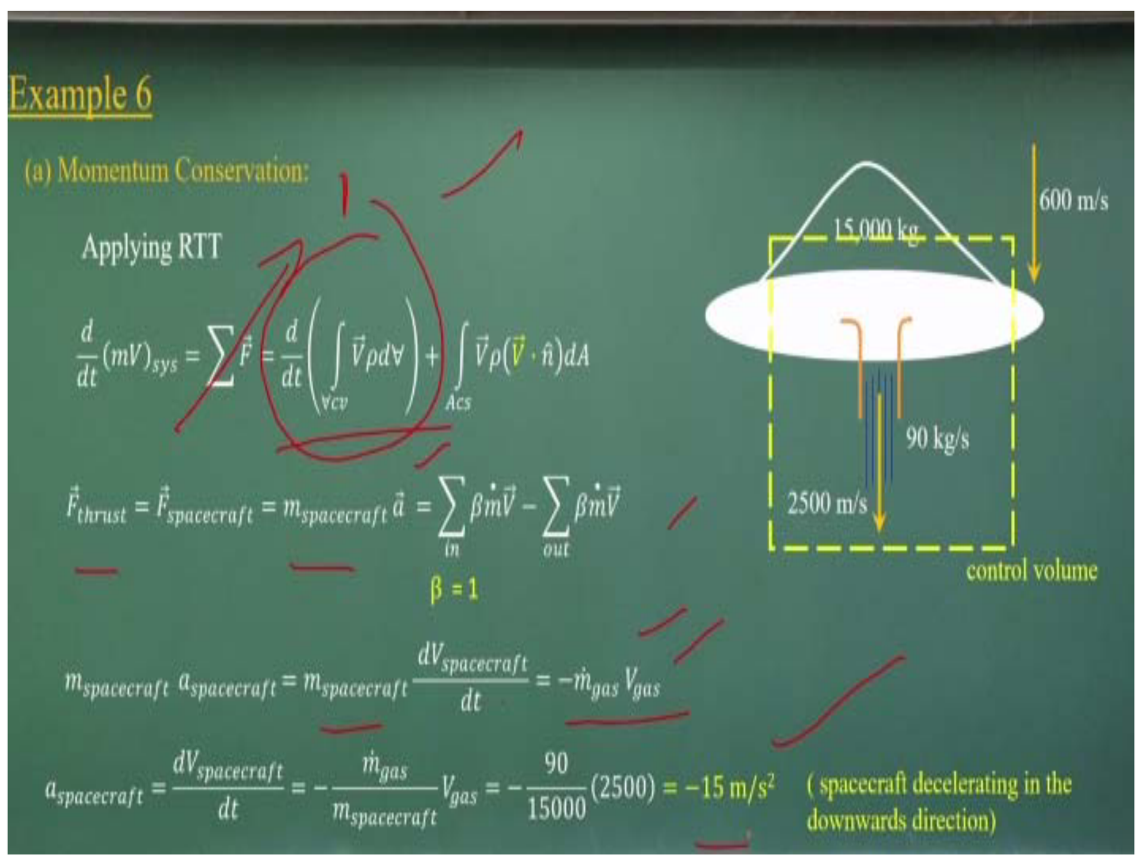

And let us apply the Reynolds transport theorems, okay. In this case, as we have considered earlier, okay. So there is this force component is zero. Only the change of the rate of the momentum flux within the control volume that what will force thrust, that will be the mass of aircraft and the accelerations. That what will be net outflux of momentum flux what is going out from this. There is no inflow.

Applying RTT

$$\frac{d}{dt}(mV)_{sys} = \sum \vec{F} = \frac{d}{dt}\left(\int_{\forall cv} \vec{V}\rho d\forall\right) + \int_{Acs} \vec{V}\rho(\vec{V}\cdot\hat{n})dA$$

$$\vec{F}_{thrust} = \vec{F}_{spacecraft} = m_{spacecraft}\,\vec{a} = \sum_{in}\beta\dot{m}\vec{V} - \sum_{out}\beta\dot{m}\vec{V}$$

$$\beta = 1$$

$$m_{spacecraft}\,a_{spacecraft} = m_{spacecraft}\frac{dV_{spacecraft}}{dt} = -\dot{m}_{gas}\,V_{gas}$$

$$a_{spacecraft} = \frac{dV_{spacecraft}}{dt} = -\frac{\dot{m}_{gas}}{m_{spacecraft}}V_{gas} = -\frac{90}{15000}(2500) = -15\text{ m/s2}$$

So you will have a because of gas injections you have a momentum flux what is m dot V gas. That what is the will be the spacecraft mass into the acceleration. This way we can compute it the acceleration will be -15 per second square. So this is the what the accelerations of the space aircraft in the negative directions. That what is decelerating in downward direction.

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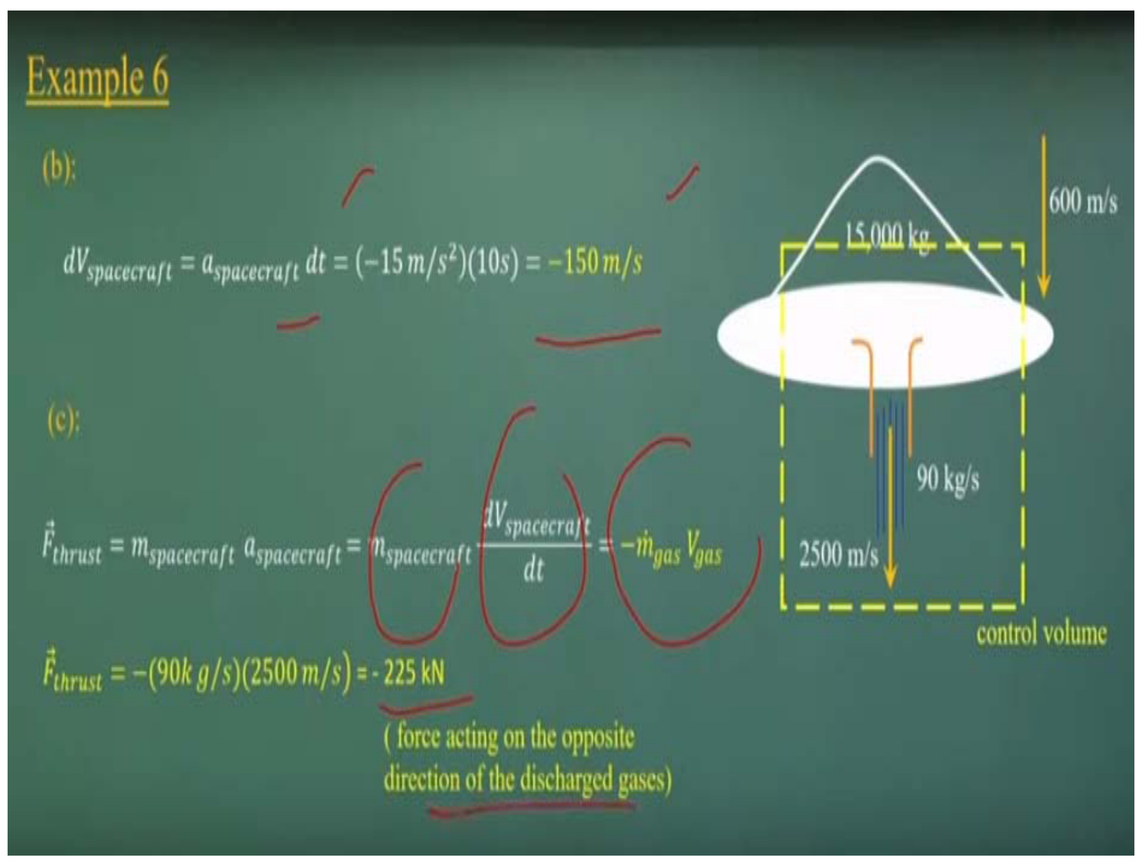

Second part of the problem is that compute the change of the velocity.

$$dV_{spacecraft} = a_{spacecraft}\,dt = (-15\,m/s^2)(10s) = -150\,m/s$$

And what will be the force acting on that which is very easy to compute and the momentum flux or mass of aircraft and the acceleration we can compute what is the thrust is acting on these,

$$\vec{F}_{thrust} = m_{spacecraft}\, a_{spacecraft} = m_{spacecraft}\frac{dV_{spacecraft}}{dt} = -\dot{m}_{gas}\, V_{gas}$$

$\vec{F}_{thrust} = -(90k\, g/s)(2500\, m/s)$ = - 225 kN

this is what will come it to 225 kilo Newton force acting opposite directions of discharged gases, that is what happens it.

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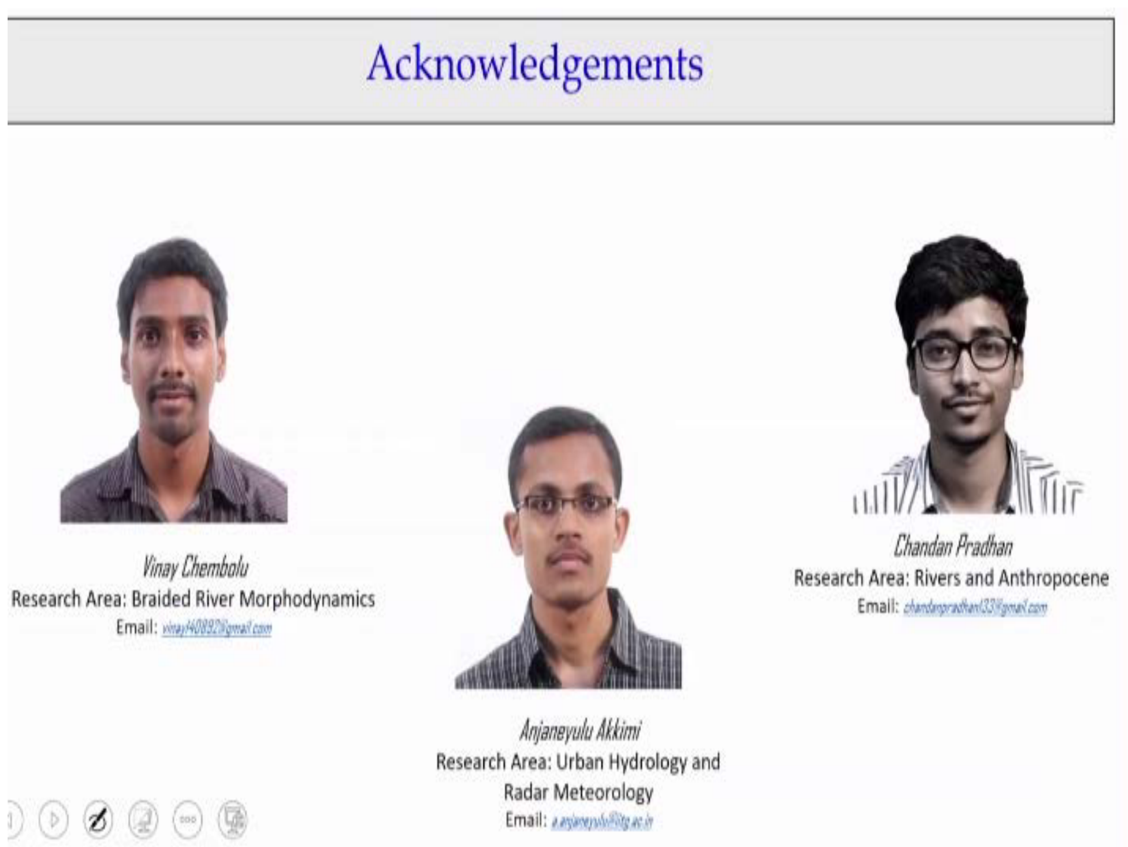

And let me acknowledge before ending this course is that my 3 PhD students they have been working hard, drawing the figures, illustrating the figures, bringing lot of examples, discussing lot before presenting you. So I do acknowledge their efforts and not to improve the knowledge of the fluid mechanics but teach the fluid mechanics in simpler form. That is what is possible because of them. I do again acknowledge their help. Okay, with this let us conclude today lectures. Thank you lot.